EUROPEAN JOURNAL OF MEDICAL AND EDUCATIONAL TECHNOLOGIES
https://www.ejmets.com
ISSN: 2732-4109 (Online)

To cite this article: Rahmoune A. On the Stabilization of the Spread of the Coronavirus (COVID-19) Pandemic in the World. European Journal of Medical and Educational Technologies 2020; 13(1): em2004. https://doi.org/10.30935/ejmets/8015

Original Article

On the Stabilization of the Spread of the Coronavirus (COVID-19) Pandemic in the World

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Received: 18 April 2020 Accepted: 21 April 2020

ABSTRACT

At the end of December 2019, a novel coronavirus (2019-Cov) emerged in China specifically it appeared in the city of Wuhan, and it has spread to the entire world very fast and in a very short time, and they posed an international public health emergency in a couple of weeks, and has attracted global attention and attained recently the position of a very high-risk category by world health organization (WHO).

This research concerns to investigate the course of the pandemic by mathematical modeling based on the information that the time-dependent change (spreading) rate of the H number of individuals who have caught a contagious disease is proportional to the multiplication of the numbers of those who have caught the disease in time delay and those who have not. According to the results of the mathematical modeling in our study, the course of the pandemic to be stable in the near time.

Keywords: COVID-19, time-varying delay, energy decay result

INTRODUCTION

It has spread COVID-19 to the entire world very fast and in a very short time, that caused thousands of people infected in the world by the respiratory tract infection, his first appearance was at the end of 2019 in the city of Wuhan in the province of Hubei in the People's Republic of China.

The diagnosis of COVID-19 is made by reverse transcription-polymerase chain reaction (RT-PCR) tests. With the help of Chinese scientists sharing genomic information on the virus fast with the entire world, it has become possible to conduct RT-PCR analysis without the virus completely spreading in the world. COVID-19 RT-PCR analysis has provided a significant opportunity in fighting with the pandemic that has emerged, which led to a marked decrease in the infections.

However, according to mutual opinions from the laboratories, some patients may have negative RT-PCR analysis results but positive clinical findings. In these cases, the RT-PCR analysis results may turn out positive in the following days.

Therefore, in the diagnosis of the disease, clinical findings are as important as laboratory findings. Additionally, in routine biochemical analysis, increases in the serum C-reactive protein (CRP), lactate dehydrogenase (LDH),

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erythrocyte sedimentation rate (ESR), monocyte volume distribution width (MDW) and D-dimer levels and a decrease in albumin concentration may be expected (see [8, 7, 6, 9, 10] and references therein).

Due to the gravity of the COVID-19 pandemic, the necessity of mathematical modeling is clear in terms of being able to make medical planning and see the long-term course of the pandemic.

In the literature, for a region with a population of p (city, country), it is stated that "the time-dependent change (spreading) rate of the H, number of individuals who have caught a contagious disease is proportional to the multiplication of the numbers of those who have caught the disease and those who have not" ([4]). The mathematical model of this situation is given by the following differential equation,

$$\frac{dH}{dt} = \mu p H(t) - \mu H^2(t) \tag{1}$$

The model above was revised specifically for the "corona pandemic" in [3], were the authors discussed it in the form of "the spreading rate of the disease ($\frac{dH}{dt}$), is directly proportional with the multiplication of the numbers of those who have caught the disease and those who have not, and inversely proportional with the square root of time-variable, were $\frac{dH}{dt}$ derivative corresponding to the time-dependent change (spreading) rate of the disease, μ is a parameter that covers all factors that influence the spreading rate, when the authors take $\mu = \mu(t) = \frac{k}{2\sqrt{t}}$ dependent on the t-time variable instead of the constantμ on the right-hand side of (1), and they give an expect the number of patients in the near time.

This research concerns to investigate the course of the pandemic by mathematical modeling based on the information that the time-dependent change (spreading) rate of the H number of individuals who have caught a contagious disease is proportional to the multiplication of the numbers of those who have caught the disease in time delay and those who have not.

The purpose of the study is, in the model given in (Eq.1), to take "the time-varying delay-dependent change (spreading) rate of the H, number of individuals who have caught a contagious disease is proportional to the multiplication of the numbers of those who have caught the disease in time delay and those who have not, we are proposing the following initial value problem related to the issue as

$$\frac{dH}{dt} = \mu_1 p H\big(t - \tau(t)\big) - \mu_2 H^2(t), t \in (0, +\infty), \tag{2}$$

$$H(0) = H_0, \tag{3}$$

$$H\big(-\tau(0)\big) = j_0\big(-\tau(0)\big). \tag{4}$$

where, μ_1, μ_2 are fixed real constants, with $\mu_1 > 0$, $\mu_2 \neq 0$, $\tau(t) > 0$ represents the time delay.

t: Independent time variable in units of days.

$H(t)$: Dependent variable expressing the number of patients at the time t,

$H\big(t - \tau(t)\big)$:Dependent variable expressing the number of patients at the paste time $t - \tau(t)$,

$\frac{dH}{dt}$: Derivative corresponding to the time-dependent change (spreading) rate of the disease,

μ_1: A parameter of delay,

μ_2: A parameter that covers all factors that influence the spreading rate.

It is well- known that the above model is not stable in absence of delay, that is if $\tau(t) = 0$, see [3]

The main objective of the present work is to establish a decay result of the (spreading) rate of the H.

Furthermore, let the initial ($t = 0$) number of patients be $H(0) = H_0$, the number of patients at a time $t = -\tau(0)$ (the past time), given as additional information be $H(-\tau(0)) = j_0$ and the number of individuals who are open to the disease be p.

On the function τ we assume that there exist positive constants τ_0, τ_1 such that

(H1) Hypotheses on $\boldsymbol{\tau}$. For the time-varying delay τ, we assume as in [1, 2] that $\tau \in W^{2,\infty}([0, T]), \forall T > 0$ and there exist positive constants τ_0, τ_1 and d satisfying

$$\begin{gathered} 0 < \tau_0 \leq \tau(t) \leq \tau_1, \tau'(t) \leq d < 1, \forall t > 0, \\ \text{and } |\mu_2| < \frac{1}{p}\sqrt{1-d}\,\mu_1. \end{gathered} \tag{5}$$

(H2) Hypotheses on $\boldsymbol{\mu_1}, \boldsymbol{\mu_2}$. For ξ and λ are positive constants, the parameter of spreading and the delay satisfy

$$\frac{p\mu_1}{\sqrt{1-d}} < \xi < 2\mu_2 - \frac{p\mu_1}{\sqrt{1-d}}, \lambda < \frac{1}{\tau_1}\left| ln \frac{p\mu_1}{\xi\sqrt{1-d}} \right|. \tag{6}$$

In order to deal with the delay feedback term, motivated by [11,1], we introduce the following new dependent variable,

$$z(\rho,t)=H(t-\tau(t)\rho),(\rho,t)\in(0,1)\times(0,\infty).$$

By computation, we have

$$\tau(t)z_t+(1-\tau'(t)\rho)z_\rho=0\ \mathrm{in}(0,1)\times(0,\infty).$$

Therefore, problem (2)-(4) can be transformed into

$$\frac{dH}{dt}=\mu_1 pz-\mu_2 z^2(0), \tag{7}$$

$$H(0)=H_0, \tag{8}$$

$$\tau(t)z_t+(1-\tau'(t)\rho)z_\rho=0, \tag{9}$$

$$H(-\tau(0))=j_0(-\tau(0)). \tag{10}$$

DECAY OF SOLUTION

In this section, we shall investigate the asymptotic behavior of the energy function E. For this, we construct a Lyapunov functional L equivalent to E, with which we can show the desired result given by Theorem 3.3. Let define the modified energy functional E associated with problem (7)-(10) by

$$E(t)=\frac{1}{2}H^2(t)+\frac{\xi}{2}\int_{t-\tau(t)}^{t}e^{\lambda(s-t)}H^2(s)\mathrm{d}s.$$

Note that, from (6), such a constant ξ exists.

The following lemma shows that the associated energy of the problem under the condition $|\mu_1|<\frac{1}{p}\sqrt{1-d}\mu_2$ is decreasing.

Lemma 2.1 *Let (H,z) be the solution of (7)-(10). Then, for some two positive constants β_1 and β_2, we have*

$$\begin{aligned}E'(t)=&\ -\beta_1|H(t)|^2-\beta_2|H(t-\tau(t))|^2\\&-\frac{\lambda\xi}{2}\int_{t-\tau(t)}^{t}e^{\lambda(s-t)}H^2(s)ds+\frac{4}{27}\mu_2\end{aligned} \tag{11}$$

Proof. Multiplying the third equation (9) by $\zeta f(z)=\zeta ze^{-\lambda\tau(t)\rho}$, and integrating over $(0,1)$ with respect to ρ, we have

$$\xi\tau(t)\int_0^1 z_t f(z(\rho,t))\mathrm{d}\rho=-\xi\int_0^1(1-\rho\tau'(t))\frac{\partial}{\partial\rho}F(z(\rho,t))\mathrm{d}\rho.$$

Thus

$$\begin{aligned}&\frac{d}{dt}\left(\frac{\xi}{2}\tau(t)\int_0^1 F(z(\rho,t))d\rho\right)\\=&\ -\frac{\xi}{2}\int_0^1\frac{\partial}{\partial\rho}[(1-\rho\tau'(t))F(z(\rho,t))]d\rho\\=&\ \frac{\xi}{2}(F(z(0,t))-F(z(1,t)))+\frac{\xi}{2}\tau'(t)F(z(1,t))\\=&\ \frac{\xi}{2}H^2(t)-\frac{\xi}{2}(1-\tau'(t))F(z(1,t))\end{aligned} \tag{12}$$

Where $F(z)=\frac{1}{2}z^2e^{-\lambda\tau(t)\rho}$.

Now, multiplying the first equation of (2) by H, integrating over Ω and exploiting the third equation (9) in above system, and multiplying the third equation by $\zeta f(z)$, and integrating over $(0,1)$ with respect to ρ, making use of (12) we get

$$\begin{aligned}E'(t)=&\ H(t)\frac{dH}{dt}-\frac{\lambda\xi}{2}\int_{t-\tau(t)}^{t}e^{\lambda(s-t)}H^2(s)ds\\&+\frac{\xi}{2}H^2(t)-\frac{\xi}{2}(1-\tau'(t))e^{-\lambda\tau(t)}H^2(t-\tau(t))\\=&\ \mu_1 pH(t)z(1,t)-\frac{\lambda\xi}{2}\int_{t-\tau(t)}^{t}e^{\lambda(s-t)}H^2(s)ds\\&-\mu_2H^3(t)+\frac{\xi}{2}H^2(t)-\frac{\xi}{2}(1-\tau'(t))e^{-\lambda\tau(t)}H^2(t-\tau(t))\end{aligned} \tag{13}$$

By young's inequality, we have

$$\mu_1(z(1,t))H(t)\le\frac{|\mu_1|}{2\sqrt{1-d}}|H|^2+\frac{|\mu_1|\sqrt{1-d}}{2}|H(t-\tau(t))|^2 \tag{14}$$

and

$$-\mu_2H^3(t)\le-\mu_2H^2(t)+\frac{4}{27}\mu_2$$

By (5), we get

$$-\frac{\xi}{2}e^{-\lambda\tau(t)}H^2(t-\tau(t))(1-\tau'(t))\le-\frac{\xi}{2}e^{-\lambda\tau_1}(1-d)H^2(t-\tau(t))$$

therefore, from (13) we get

$$\begin{aligned}E'(t)=&\ -\left(\mu_2-\frac{\xi}{2}-\frac{|\mu_1|p}{2\sqrt{1-d}}\right)|H(t)|^2\\&-\left(\frac{\xi(1-d)}{2}e^{-\lambda\tau_1}-\frac{|\mu_1|p\sqrt{1-d}}{2}\right)|H(t-\tau(t))|^2\\&-\frac{\lambda\xi}{2}\int_{t-\tau(t)}^{t}e^{\lambda(s-t)}H^2(s)\mathrm{d}s+\frac{4}{27}\mu_2\end{aligned}$$

Letting $\beta_1=\mu_2-\frac{\xi}{2}-\frac{|\mu_1|p}{2\sqrt{1-d}}>0$ and $\beta_2=\frac{\xi(1-d)}{2}e^{-\lambda\tau_1}-\frac{|\mu_1|p\sqrt{1-d}}{2}>0$, this completes the proof of Lemma 2.1.

GENERAL DECAY

In this section, we shall investigate the asymptotic behavior of the energy function E. For this, we construct a Lyapunov functional L equivalent to E, which we can show the desired result. First, we define some functionals and establish several lemmas. Let

$$L(t) = ME(t) + \Psi(t) \tag{15}$$

where M is positive constant to be choose later, and

$$\Psi(t) = \frac{1}{2}H^2(t)$$

The functional L is equivalent to the energy function E by the following lemma.

Lemma 3.1 *For $M > 0$ large enough, there exist two positive constants C_1 and C_2 such that,*

$$C_1E(t) \le \mathrm{L}(t) \le C_2E(t), t \ge 0.$$

Proof. By (15), we have

$$|\mathrm{L}(t) - ME(t)| = \frac{1}{2}|H^2(t)| \le CE(t)$$

Thus, we obtain

$$(M - C)E(t) \le L(t) \le (M + C)E(t)$$

Choosing $M > 0$ large, the proof of Lemma 3.1 is concludes.

Lemma 3.2 *For $M > 0$ large enough, there exist positive constants C_3, C_4 and $t_0 > 0$ such that*

$$L'(t) \le -C_3E(t) - C_4E'(t) + \frac{4}{27}(C_4 + M)\mu_2, t > 0 \tag{16}$$

Proof. Using problem (2)-(4), and (14) we have

$$\begin{aligned} \psi'(t) = \quad & H(t)\frac{dH}{dt} = \mu_1 H(t)H\big(t-\tau(t)\big) - \mu_2 H^3(t) \\ \le \quad & \frac{|\mu_1|p}{2\sqrt{1-d}}|H(t)|^2 + \frac{|\mu_1|p\sqrt{1-d}}{2}|H(t-\tau(t))|^2 - \mu_2H^2(t) + \frac{4}{27}\mu_2 \end{aligned} \tag{17}$$

Therefore

$$\begin{aligned} \mathrm{L}'(t) \le \quad & -\left(M\beta_1 + \mu_2 - \frac{|\mu_1|p}{2\sqrt{1-d}}\right)|H(t)|^2 - \frac{\lambda M\xi}{2}\int_{t-\tau(t)}^{t} e^{\lambda(s-t)}H^2(s)\mathrm{d}s \\ & -\left(M\beta_2 - \frac{|\mu_1|p\sqrt{1-d}}{2}\right)|H(t-\tau(t))|^2 + \frac{4}{27}M\mu_2 \end{aligned}$$

then choose $M > 0$ so large that

$$M > \max\left(\frac{|\mu_1|p}{2\beta_1\sqrt{1-d}}, \frac{|\mu_1|p\sqrt{1-d}}{2\beta_2}\right),$$

we complete the proof of Lemma 3.2.

Now, we are ready to give and prove the main theorem.

Theorem 3.3 *There exist positive constants C_0, C_5, C such that*

$$E(t) \le C_0e^{-Ct} + C_5, \mathrm{for} t > 0.$$

Proof. From (16), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{L}(t) + C_4E(t)) \le -C_3E(t) + \frac{4}{27}(C_4 + M)\mu_2, \mathrm{for}\ t > 0.$$

Let define

$$\Phi(t) = \mathrm{L}(t) + C_4E(t)$$

then we can easily see that $\Phi(t)$ is equivalent to $E(t)$. Hence, we arrive at

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi(t) \le -CE(t) + \frac{4}{27}(C_4 + M)\mu_2, t > 0$$

Integrating this over $(0, t)$, we deduce that

$$\Phi(t) \le \frac{4}{27C}(C_4 + M)\mu_2 + \left(\Phi(0) - \frac{4}{27C}(C_4 + M)\mu_2\right)e^{-Ct}, t > 0$$

This completes the proof from the equivalent relation of $\Phi(t), \mathrm{L}(t)$ and $E(t)$.

Remark 3.4

1. Then for t so large enough, the number of patients settle down at value $\frac{4}{27C}(C_4 + M)\mu_2$.

2. We can give exactly the values of all constants.

ACKNOWLEDGEMENT

The author would like to thank the editor of European Journal of Medical and Educational Technologies. His precious comments have greatly contributed to improving the quality of the paper.

DECLARATION OF CONFLICT OF INTEREST

The authors received no financial support for the research and/or authorship of this article. There is no conflict of interest.

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