




■ Original Article

Numerical Prediction for Spreading Novel Coronavirus Disease in India Using Logistic Growth and SIR Models

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ABSTRACT

At present, Novel COVID-19 has become the greatest issue in the world which was first detected in the city of Wuhan of Hubei province in China in the month of December 2019. SARS-COV-2 is responsible for the spreading of corona virus disease. Within a very short time period, it has spread very fast throughout the world. Beyond all the boundaries of medical science, nowadays COVID-19 has become a main interesting topic in many research fields such as Applied Mathematics, economy, politics, up to the living room. The aim of this study is to investigate the dynamic behavior of pandemic COVID-19 which based on real-time data. The logistic growth model and SIR model has been employed to study the different four characteristics of COVID-19, such as low growth state, moderate growth state, transition state, and steady-state. The models have been validated with the results of real-time data. Moreover, the model presents a rapid change due to the unavailability of precautions. Furthermore, some parameters have been implemented to predict the COVID-19 status up to 5th Jan 2021. From these models, it is predicted that the total number of infected peoples reaches 10M up to 5th Jan 2021. It has also been revealed that with the support of lockdown, social alertness, increasing testing facility, and social distancing recovery growth rate of infected persons increases with the increase of time.

Keywords: COVID-19, logistic growth model, SIR model, Novel coronavirus

INTRODUCTION

In December 2019, the novel coronavirus was first detected in Wuhan, Hubei Province, China, [1, 2], which has become the largest issue in the world. SARS-COV-2 is responsible for the spreading of corona virus disease. It has spread very fast throughout the world within a very short time period. Nowadays, COVID-19 has become a main interesting topic in many research fields such as Applied Mathematics, economy, politics, up to the living room beyond all the

boundaries of medical science. On 30 January 2020, the World Health Organization [10-19] declared the epidemic of an international public health crisis and 11th march declared as a pandemic situation. As of 1st June 2020, in over 188 countries and territories, over 6.15 million cases of COVID-19 are identified. During this pandemic, already 371000 people have died [3-5]. Moreover, in worldwide 2.80M peoples have been recovered from this virus. Fever, tiredness, tiredness, shortness of breath and loss of sense of smell are the typical symptoms of this infection. Pneumonia and acute

respiratory distress syndrome were found in infected people in some cases. Moreover, in some cases, no such type of symptoms has been observed in infected persons. According to medical science, the general symptoms are reflected after five days in infected persons. Generally, the corona virus has spread due to close contact of people often via coughing and sneezing.

On 30th January, the first COVID-19 case was observed in India. Until 25 June, 491970 infected cases, and 15319 deaths were identified In India [6-8]. However, 285,983 persons have been recovered. The Ministry of Health stated that India has become one of the world's lowest death rates registered of COVID-19 over the world. The maximum number of infected cases was identified in six cities such as Mumbai, Delhi, Ahmadabad, Chennai, Pune and Kolkata respectively. Government has provided some guidance for peoples to reduce the possibility of infectious disease with COVID-19, including washing hands, trying to cover mouth when one's coughing, establishing good distance, self-isolation, wearing gloves in social places. Till 25th June there is no such vaccines have found to recover the disease. All over the world, all the medical researchers are trying to find the out vaccine.

The seriousness of the COVID-19 pandemic situation makes it clear that scientific modeling is very much helpful to carry out medical planning and monitor the pandemic's long-term courses. The main objective of this study is to investigate the spreading rate of infected disease in the next two months. Logistic growth and SIR models have been implemented to understand the status of the pandemic situations by analyzing the spreading rate, recovery rate, and death rate respectively. Four basic characteristics of COVID-19, such as low growth state, moderate growth state, transition state and steady-state are evolved to better understand the present situation and the situation up to 5th Jan 2021. The governing ordinary differential equations are solved, and simulations have has done with different kinds of parameters which are estimated by utilizing MATLAB and MATHEMATICA [24]. The present models have been validated with the results of real-time data [3-9]. The results of this mathematical modeling will assist the relevant authorities in preparing adequate medical plans and taking the necessary steps to manage the pandemic situation.

MATHEMATICAL MODELS

The present pandemic situations and future predictions of infected growth rate, recovery rate, and death rate have been analyzed by logistic growth rate and SIR models. The models are described as below sections:

Logistic Growth Model

Logistic growth model can be used to design the infected Population growth which was first introduced by Pierre-Francois Verhulst pioneered in biological systems. Frequency of spread or new infection is proportional to the number of total infected peoples and the number of peoples who have not encountered an infectious disease. Therefore, the current pandemic situation is governed by the following time dependent ordinary differential equation.

$$\frac{dC}{dt} = LC(N - C) \quad (1)$$

Here, C, t, and N denoted as the total active cases, time (day), total population of India respectively. Whereas L be the parameter which noted as the spreading rate of COVID-19 and the new active cases/day can be represent as $\frac{dC}{dt}$. This parameter L has been used as a time dependent parameter to make the model more compatible with the real-time data. Due to social awarness and Government activities the rate of spread will decrease with the time. Hence L can be represented as $L = \frac{C_p}{2\sqrt{t}}$, l be the proportional constant. Initially it is considered that at t=0, C(0)= C₀. In this study, t= 0 be taken on 4th March, 2020, and after 196 days at t= t₁ the number of infected persons will be C (t₁)= C₁ as on the date up to 5th Jan 2021. Therefore, the model can be modified as in the following form of equation:

$$\frac{dC}{dt} = \frac{C_p}{2\sqrt{t}} C(N - C) \quad (2)$$

$$C(t = 0) = C_0 \quad (3)$$

$$C(t = t_1) = C_1 \quad (4)$$

After solving the equation (2), we get the following form:

$$C(t) = \frac{Nf}{f + e^{-NC_p\sqrt{t}}} \quad (5)$$

Here $f = \frac{C_0}{(N-C_0)}$, be the constant. The equation (5) with the the conditions (3), and (4) we have,

$$C(t) = \frac{N}{1 + \frac{(N - C_0)}{C_0} e^{-NC_p\sqrt{t}}} \quad (6)$$

Whereas,

$$C_p = \frac{1}{N\sqrt{t_1}} \ln \left(\frac{C_1(N - C_0)}{C_0(N - C_1)} \right) \quad (7)$$

In this study, $C_0= 29$ (4th March) and $C_1= 10,357,569$ (up to 5th Jan 2021), whereas $t_1= 196$ days. Therefore, it is seen that the control parameter (C_p) depends on time, constant population and C_1 and C_0 . Here $N= 13,826,42280$ as per real-time data [6]. Moreover, we have calculated the $\frac{d^2C}{dt^2}$ to know the dynamic of increasing growth rate/day. Hence, from the equations [2-7], we get the following form:

$$\frac{d^2C}{dt^2} = \frac{C_p}{2\sqrt{t}}(N - 2C) - \frac{1}{2t} \frac{C_p}{2\sqrt{t}} C(N - C) \quad (8)$$

In addition, we considered a simple ordinary differential equation in terms of observing the phenomena of recovery and death per day, for the 15 days and up to 5th Jan 2021. Evidently, the death rate is proportional to the number of infected persons. Therefore the general equation for the death rate is as follows:

$$\frac{dD}{dt} = \frac{h}{2\sqrt{t}} C(t) \quad (9)$$

$$D(t = 0) = D_0 \quad (10)$$

Here h is the proportional constant defined as control parameter and $D(t)$ can be defined as the total number of death in a region or country. In this case $D_0= 10$ (24th March) be taken as per real-time data. Furthermore, the recovery rate is also proportional to the total infected persons which governed by the following equation:

$$\frac{dR}{dt} = \frac{r}{2\sqrt{t}} C(t) \quad (11)$$

$$R(t = 0) = R_0 \quad (12)$$

Whereas $R(t)$ can be defined as the total number of deaths in a region or country. In this case $R_0= 6$ (24th March) be taken as per real-time data. To observe the fact about the number of infected persons, death rate, recovery rate we have taken the epidemiological SIR model to introduce the comparing results of infected persons and recovery rate.

SIR Model

SIR model stands for Susceptible-Infected-Recovered which is a compartmental disease model to investigate the disease spread in a population of region or country [20-23]. In this present study the constant population belongs to any one

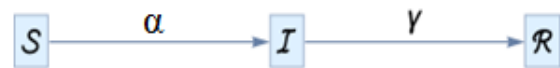


Figure 1. Schematic diagram of present situation

of the three compartments: Susceptible, Infected or Recovered. Almost 100 years ago SIR model were first developed by Kenmark and McKendrick. Schematic diagram (**Figure 1**) of this model prescribed as below:

The arrow indication indicates that the relation between one state to another state with the rate of movement. Whereas α , can be noted as transmission rate coefficient. However, the recovery rate constant is defined as $\gamma= 1/t$. The model states that when susceptible persons come in close contact with infected persons ($I(t)$) then they become also infected and after some period of time they recover. The total number of population is defined as $N= S(t)+I(t)+R(t)$, which are formulized by the following two-dimensional autonomous differential equations:

$$\frac{dS}{dt} = -\alpha S(t)I(t) \quad (13)$$

$$S(t = 0) = S_0 \quad (14)$$

$$\frac{dI}{dt} = \alpha S(t)I(t) - \gamma I(t) \quad (15)$$

$$I(t = 0) = I_0 \quad (16)$$

$$\frac{dR}{dt} = \gamma I(t) \quad (17)$$

$$R(t = 0) = R_0 \quad (18)$$

The following assumptions have been considered for further studies:

- The total of population being constant during the phase of modeling.
- Each n every infected person has the same opportunity to be recovered.
- Other infections are not considered in this model.
- The real-time data are taken from the Ministry of Health welfare website.

In India’s current situation the above initial values have been taken from the real time data, such as $S_0= 510672$ (15th Feb to 26th June), $I_0= 197840$, $R_0= 296376$ and the corresponding

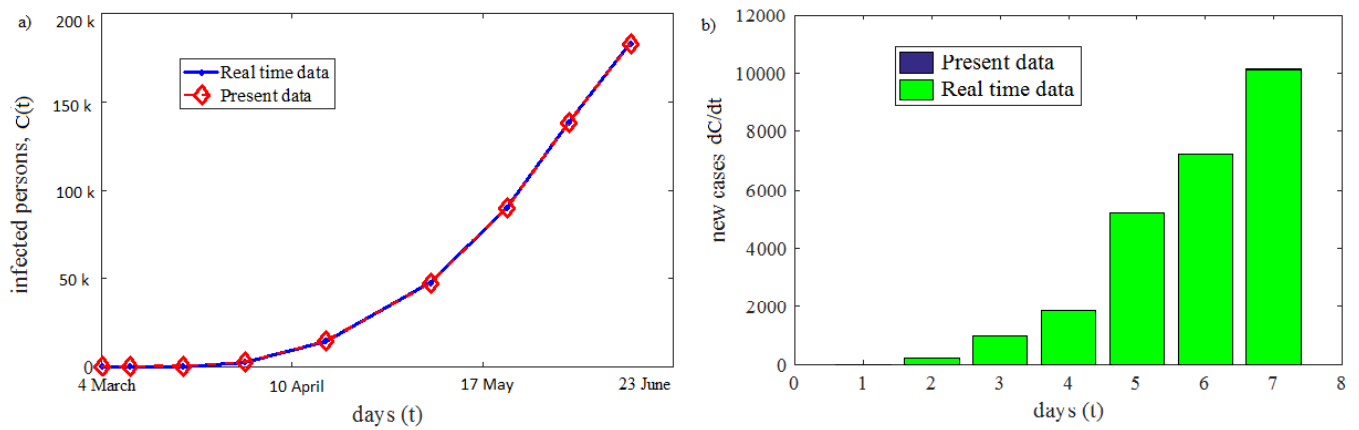


Figure 2. Infected person upto 23rd June, 2020 [a], new cases average per 15 days [b]

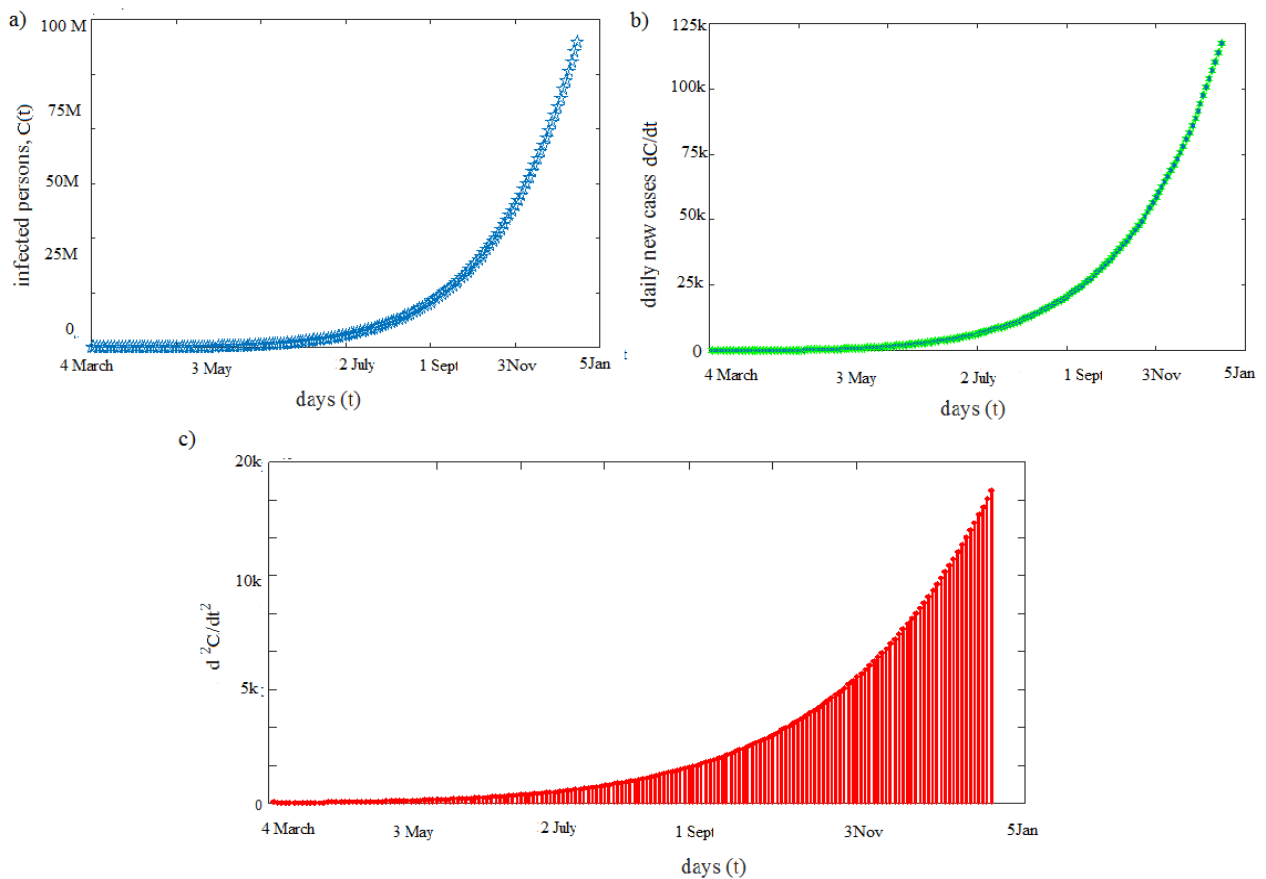


Figure 3. Infected person [a], daily new cases [b], and number of daily growth rate [c] vs time

parameters are taken as $\alpha = \gamma/S_0$, $\gamma = 1/128$ (27th June to up to 5th Jan 2021). From these values anyone can track the daily growth rate and recovery rate of the affected people.

MODEL VALIDATION

The present results of Logistic growth model have been validated with the results of real-time data. New cases fluctuated every day as per the previous day that’s why bar diagram is used for more clarity. Two validations (**Figure 2a-2b**) are given as below to make the good relationship between the logistic model and real time data results.

RESULTS AND DISCUSSIONS

In this section we will analyze in detail the infected rate, death rate and recovery rate using the Logistic model with the corresponding governing equations (1-11). Infected rates with the variation of time are shown in **Figure 3a**. From **Figure 3a**, it is clear that if peoples don’t follow the Government rules and suggestions the number of infected people will reach about 10M on up to 5th Jan 2021 which is approximately 344,827.6 times of the initial date (4th March).

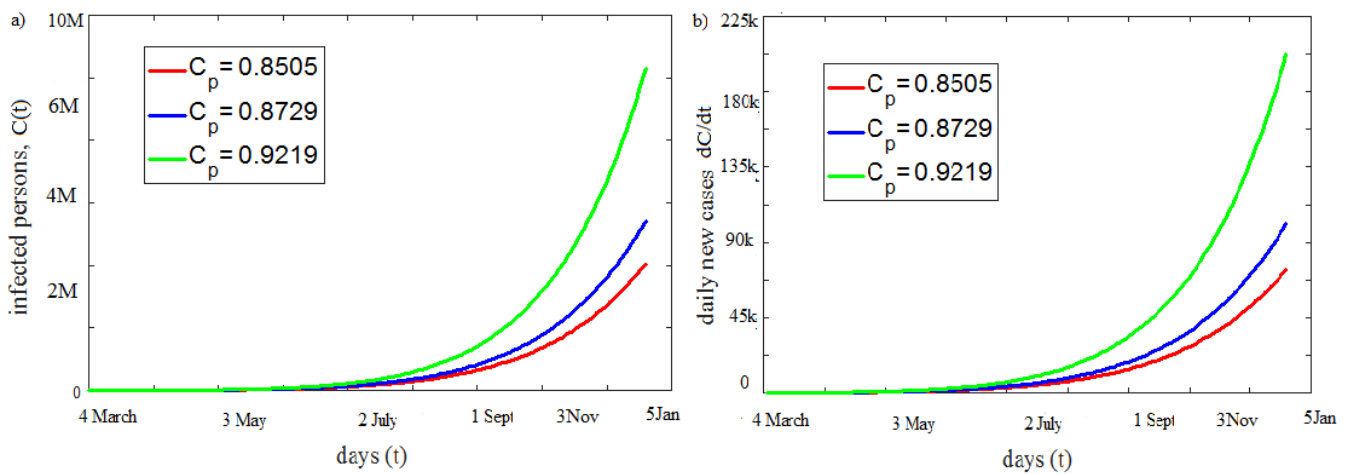


Figure 4. Effect of C_p on total number of infected persons [a] and daily new cases [b] up to 5th Jan, 2021

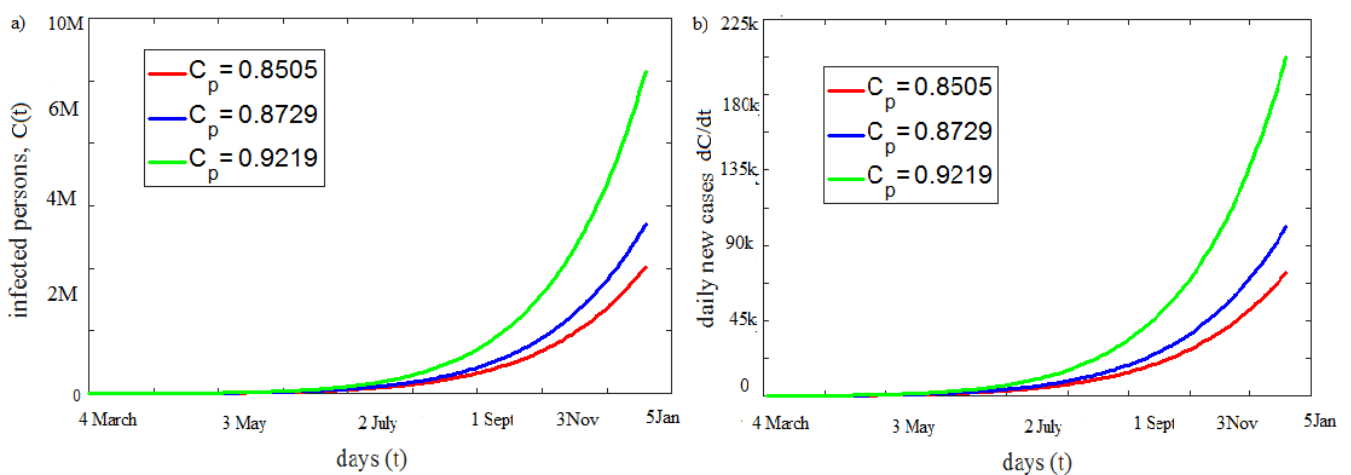


Figure 5. Effect of the parameter h on total number of expected death persons [a] and r on total number of recovery persons [b] up to 5th Jan, 2021.

Moreover, **Figure 3b** present the daily infected rate with the increase of time. It is demonstrated that infected rate increases per day as compared to previous day. According to the model, it is concluded that if this pandemic situation in India does not improve then daily new case may raise up to 95000 on up to 5th Jan 2021. Number of daily growth rate is shown in **Figure 3c**. The effect of control parameter (C_p) is also discussed in this study to better understand the pandemic situation on next two months. Moreover, from the **Figure 4a-4b** it has been observed that the increase in (C_p) will lead to an increase in daily cases and total cases in the next two months.

Equation 5 presents the inverse relationship between C_p and time, (t). Hence it has also been observed that as time decreases the value of (C_p) increases therefore infected rates also increases and vice versa. **Figure 5a** show the effect of death parameter (h) on the computed total deaths ($D(t)$). It is observed that a slightly change in h causes the reduce in total death significantly. Here, $h=1$ be the current situation.

Due to medical facilities in the affected areas and various government activities, it is predicted that the level of h will decrease somewhat. So, when h goes to zero it means the total number of deaths will be determined when suitable vaccine or drug is available. In addition, recovery rate also predicts from the effect of recovery parameter (r) which is shown **Figure 5b**. Here $r = 0.56$ (26th June) be the present situation. It is also observed that total recoveries increase when the value of r increases with the support of medical facilities and all other Government activities. To making more compatible with the present situation we have further discussed the infected and recovery rates by the SIR model via phase plane analysis as follows.

Conserved Quantity

A conserved quantity of a dynamical system is based on the dependent variable whose value remains constant along each trajectory of the system. In short, the variable does not

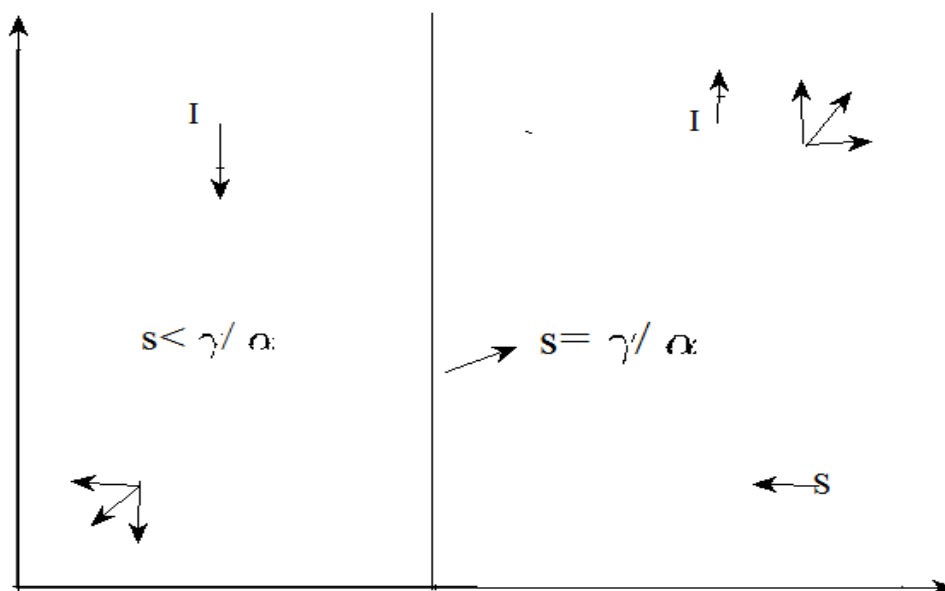


Figure 6. Schematic diagram of trajectories direction

change at any condition. Analytically we have discussed the conserved quantity in the following subsection.

Phase-plane analytical approach

Analytically we are going to discuss on how to survive in this epidemic situation. The equation 15 can be written as

$$\frac{dI}{dt} = (\alpha S(t) - \gamma)I(t) \tag{19}$$

- If $\frac{dI}{dt} > 0, (\alpha S(t) - \gamma) > 0$, then the epidemic grows significantly becomes the situation is out of controlled.
- If $\frac{dI}{dt} < 0, (\alpha S(t) - \gamma) < 0$, then the epidemic shrinks.
- Threshold= γ/α , if $S < \gamma/\alpha$, no epidemic.

We are assuming three steps to survive in this pandemic situation until medicine evolve.

Assumption 1: The value of transmission parameter (α) decreases when peoples will follow the basic steps like hand washing, elbow sneezing, sick children at home and all government activities then the epidemic situation is under controlled.

Assumption 2: The value of susceptible rates (S) will decrease when the vaccines are evolved.

Assumption 3: People recover at this stage when γ increases.

In addition, solving the equation (12) with the help of equation (14) we get the total number of susceptible persons. Therefore, we get,

$$S(t) = S_0 e^{\frac{\alpha(R_0 - R(t))}{\gamma}} \tag{20}$$

As t tends to ∞ , the equation becomes

$$S_\infty = S_0 e^{\frac{\alpha(R_0 - R_\infty)}{\gamma}} \tag{21}$$

Here R_∞ be the total number of recovered persons If we set the value of all the parameters, the total number of susceptible cases can be found. Here R_∞ be the total number of recovered persons. The total number of susceptible cases can be found by setting all the parameters value. In this study we conclude our discussion of the SIR model with conserved quantity. The equations (19-21) with the general conditions can be prescribed as in following schematic diagram (**Figure 6**).

Equations (15-17) are used to find the trajectories which are governed by the following equation (22):

$$\frac{dI}{dS} = -1 + \frac{\gamma}{\alpha S} \tag{22}$$

Taking integration on both sides we get

$$I = -S + \gamma/\alpha \log(S) + k \tag{23}$$

Here k being the constant. Using the initial conditions 14 and 16 we have,

$$k = I_0 + S_0 + \gamma/\alpha \log(S_0) \tag{24}$$

Therefore we get

$$I + S - \gamma/\alpha \log(S) = k$$

$$= I_0 + S_0 + \gamma/\alpha \log(S_0) \quad (25)$$

From the equation (25) we are able to find out the maximum number of infected persons (equation 26) for a given initial conditions.

$$I_{max} = S - S_0 + \gamma/\alpha \log(S_0/S) \quad (26)$$

Awareness and Government Activities

The COVID-19 situation in India presently emerges as outrageous. Several steps have been taken by the India Government such as social distancing, curfew, 95 days' lockdown (till 27th June), etc. Due to the economic downturn in India the Indian government is making a decision to slowly lose down the lock-down. Some special treatments have been by the Government like maintain social distancing, using mask outside home, frequently hands washing, avoid gathering etc. Most of the active cases were rises in metropolitans but now a days it infects more in ruler areas due to migrant workers are began to come at home. To protect the community there is no effective vaccine or accurate and effective therapy has not been evolved till now. So, if the situation is to be kept under control, government regulations must be adhered to until the vaccine is evolved. The pandemic situation of COVID-19 is expected to be successfully overcome by humans. We all are wish that one day our India and worldwide will free from COVID-19. As per our analysis it is studied that even a small increase in the precautions will generate a relaxation of the infected community.

CONCLUSIONS

The Outbreak COVID-19 in India was studied by Logistic growth model and SIR models. The results of Logistic growth model (infected rate and daily infected rate) have been validated with the real-time data. The concluding remarks are as follows:

- It is clearly apparent that if peoples don't follow the Government rules and suggestions the number of infected people will reach about 10M on up to 5th Jan 2021, which is approximately 344,827.6 times of the initial date (4th March). Moreover, it is studied that the daily infected rate with the increase of time. It is demonstrated that infected rate increases per day as compared to previous day.

- According to the model, it is concluded that if this pandemic situation in India does not improve then daily new case may raise up to 95000 on 5th January, 2021.
- It has also been observed that as time decreases the value of C_p increases therefore infected rates also increases and vice versa.
- It is observed that a slightly change in h causes the reduce in total death significantly. Due to medical facilities in the affected areas and various government activities, it is predicted that the level of H will decrease somewhat. So, when H goes to zero it means the total number of deaths will be determined when suitable vaccine or drug is available.
- It is also observed that total recoveries increase when the value of $r(t)$ increases with the support of medical facilities and all other Government activities.
- If sufficient precautions are not taken, or Government rules are obeyed then the pandemic situation becomes out of controlled. Consequently, it shows a very fast change in the negative direction. So, the government fixed rules are mandatory to all of us.
- It is extremely useful to treat personal hygiene in particular individually, to apply social isolation, to strengthen the immune system through natural or healthy nutrition, to promote social insulation, to scan for those who are with COVID-19 symptoms and, especially to take care the elder as well as Younger ages.
- Our model is helpful to predict dynamics of future of COVID-19 situation in India. It is also seen that the infection rate and death rate will soon increase in next two months due to lockdown relaxation, low surveillance in various places and migrant workers arriving in the villages from the Red Zone area. However, the recovery rate also increases little much but not fully. So, maintain the government rules even after the unlock period then the pandemic situation is under controlled.

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